Symmetry-Enhanced Boundary Qubits at Infinite Temperature

Jack Kemp,[1,2] Norman Y. Yao,[2] and Chris R. Laumann[3]
[1]*Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, United Kingdom*
[2]*Department of Physics, University of California, Berkeley, California 94720, USA*
[3]*Department of Physics, Boston University, Boston, Massachusetts 02215, USA*

(Received 8 February 2020; accepted 9 October 2020; published 13 November 2020)

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry-protected topological (SPT) phase hosts a robust boundary qubit at zero temperature. At finite energy density, the SPT phase is destroyed and bulk observables equilibrate in finite time. Nevertheless, we predict parametric regimes in which the boundary qubit survives to arbitrarily high temperature, with an exponentially longer coherence time than that of the thermal bulk degrees of freedom. In a dual picture, the persistence of the qubit stems from the inability of the bulk to absorb the virtual $\mathbb{Z}_2 \times \mathbb{Z}_2$ domain walls emitted by the edge during the relaxation process. We confirm the long coherence times via exact diagonalization and connect it to the presence of a pair of conjugate almost strong zero modes. Our results provide a route to experimentally construct long-lived coherent boundary qubits at infinite temperature in disorder-free systems. To this end, we propose and analyze an implementation using a Rydberg optical-tweezer array and demonstrate that the difference between edge- and bulk-spin autocorrelators can be distinguished on timescales significantly shorter than the typical coherence time.

DOI: 10.1103/PhysRevLett.125.200506

The primary signature of symmetry-protected topological (SPT) order is the presence of robust boundary degrees of freedom at zero temperature [1–8]. At *finite* temperature, these boundary modes interact strongly with thermal excitations in the bulk and rapidly decohere. Recent progress on understanding many-body localized (MBL) states of matter [9,10] has yielded the insight that such edge modes can be stabilized at finite temperature via strong quenched disorder [11–14]. In this case, the disorder serves to localize bulk thermal excitations, preventing them from scattering with and decohering the boundary mode. While intriguing, the requirement of strong disorder complicates prospects for realizing such MBL SPT phases in experiments [15–19] and also weakens the distinguishing feature of the decoupled boundary mode, since bulk transport is also arrested.

In this Letter, we describe how the boundary and bulk degrees of freedom in a translationally invariant system can decouple parametrically, even at infinite temperature. This separation of edge and bulk dynamics stems from the inability of the edge to resonantly absorb or emit bulk excitations. We exploit this dynamical protection to construct a coherent edge qubit in a one-dimensional spin chain without disorder.

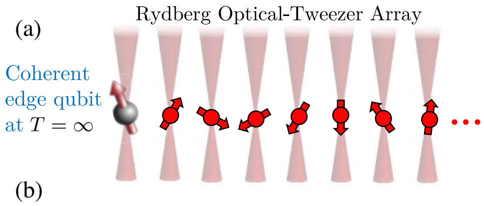
In particular, we show that the ZXZ or "cluster" model [20–22] defined on an open one-dimensional chain with $L = 2M$ sites as

$$H_{\mathrm{SPT}} = \lambda_1 \sum_{j=1}^{M-2} \sigma^z_{2j}\sigma^x_{2j+1}\sigma^z_{2j+2} + \lambda_2 \sum_{j=1}^{M-3} \sigma^z_{2j+1}\sigma^x_{2j+2}\sigma^z_{2j+3} + \Gamma \sum_{j=1}^{L} \sigma^x_j + \Gamma_2 \sum_{j=1}^{L-1} \sigma^x_j \sigma^x_{j+1} \tag{1}$$

can support a coherent edge qubit at any temperature, as long as it is dimerized with $\lambda_1 \neq \lambda_2$. The presence of this qubit owes to the existence of two long-lived, *conjugate* boundary modes, the usual example of which are $\{\sigma^z_{\mathrm{edge}}, \sigma^x_{\mathrm{edge}}\}$ (Fig. 1). Crucially, the dimerization breaks the $\mathbb{Z}_2$ swap symmetry between even and odd spins, but keeps the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry of the SPT phase intact. Finally, we propose an experimental realization of the model in a 1D Rydberg tweezer array and describe how the coherence of the edge qubit can be directly probed.

Edge decoupling of a classical bit.—Before discussing the emergence of an edge qubit, we provide some intuition for the edge-decoupling mechanism [23,24]. This mechanism can already be illustrated for the *classical* edge polarization of a quantum transverse-field Ising chain. In particular, consider the Hamiltonian

$$H_{\mathrm{Ising}} = -J \sum_{j=1}^{L-1} \sigma^z_j \sigma^z_{j+1} - \Gamma \sum_{j=1}^{L} \sigma^x_j - J_2 \sum_{j=1}^{L-2} \sigma^z_j \sigma^z_{j+2}, \tag{2}$$



(b)

Model	$T = 0$	$T = \infty$
Ising	Bit	Prethermal Bit
ZXZ	Qubit	No Edge Bit
Dimerized ZXZ	Qubit	Prethermal Qubit

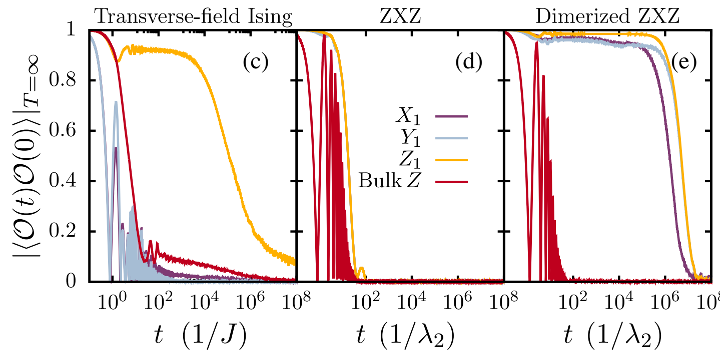


FIG. 1. (a) Schematic illustration of a one-dimensional Rydberg optical-tweezer array which hosts a coherent edge qubit even at infinite temperature. (b) Comparison of edge-mode behavior at zero and infinite temperature in each of the models discussed in the main text. (c)–(e) The autocorrelator of the edge-spin operators at infinite temperature from exact diagonalization for (c) the transverse-field Ising chain of Eq. (2) with $J_2 = \Gamma = 0.25J$, compared with (d) the ZXZ chain of Eq. (1) with $\Gamma = \Gamma_2 = 0.05$, $\lambda_1 = \lambda_2 = 1$, and (e) the dimerized ZXZ chain with $\lambda_2 = 0.6$, for system size $L = 14$.

where $\sigma^{x/z}$ are Pauli operators. It is well known that the ground state of this system (for small enough Γ) is ferromagnetic. Dynamically, this is captured by the autocorrelation of the bulk magnetization, $\langle \sigma^z(t)\sigma^z(0)\rangle_{T=0} \underset{t\to\infty}{\longrightarrow} M^2 \neq 0$.

At nonzero temperature, quantum dynamics cause bulk observables to thermalize and such long-range order is lost. In particular, $\langle \sigma^z(t)\sigma^z(0)\rangle_T$ decays as $\sim e^{-t/\tau_{\rm bulk}}$ with a timescale $\tau_{\rm bulk} \sim 1/\Gamma$ due to the propagation of bulk domain walls. Surprisingly, even at *infinite* temperature in this interacting model, the edge magnetization σ_1^z (and σ_L^z) can decay significantly more slowly [23,24]. For $\Gamma, J_2 \ll J$, $\langle \sigma_1^z(t)\sigma_1^z(0)\rangle \sim e^{-t/\tau_{\rm edge}}$ with

$$\frac{1}{\tau_{\rm edge}} \sim \Gamma\left(\frac{\Gamma}{J}\right)^{cJ/J_2}, \tag{3}$$

as can be seen over 2 orders of magnitude in Fig. 2.

To understand the enhanced stability of the edge magnetization, consider the excited states of the chain in terms of domain walls in the σ^z configuration. For $J_2 = \Gamma = 0$, domain walls cost energy $2J$. If the edge-spin σ_1^z flips, it changes the number of domain walls by ± 1 and the energy by $\pm 2J$. In the bulk, turning on a perturbative transverse-field Γ can only change the number of domain walls by $0, \pm 2$. Thus, all finite-order perturbative processes (in Γ) which depolarize σ_1^z are off resonant by at least $\Delta E = \pm 2J$ and the edge magnetization cannot decay ($\tau_{\rm edge} \to \infty$).

At finite J_2, the domain walls interact: Any pair of domain walls gains energy $2J_2$ when they are neighbors. Thus, it is possible to compensate the energy $\sim 2J$ of an extra domain wall by rearranging of order $n \sim J/J_2$ domain walls to sit next to one another. Using this as the leading-order on-shell process produces the exponential prethermal timescale of Eq. (3).

The edge-magnetization σ_1^z in the Ising model thus constitutes a long-lived *classical* bit at the boundary—it resists depolarization from bulk dynamics even at high temperature. However, it is not a long-lived *quantum* bit, which can be in a superposition of states, and thus must also resist "dephasing." Any local operator conjugate to σ_1^z (e.g., σ_1^x or σ_1^y) creates domain walls whose propagation leads to decay on a timescale $O(1/\Gamma)$; see Fig. 1(c).

Edge decoupling of a quantum bit.—Having developed intuition for the long-lived classical polarization, we turn to the edge qubit in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT phase [25]. For this qubit to remain coherent at high temperatures, the *pair* of conjugate boundary modes corresponding to σ^z and σ^x must be long-lived. The simplest way to achieve this would be to generalize the "domain-wall absorption arguments" to each conjugate edge mode of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT. Unfortunately, there is a complication: In the transverse-field Ising model, there is only a single type of domain wall, whereas in a $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT, there are multiple types of excitations, leading to many more channels for depolarization and dephasing.

The excitation structure of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT is easily understood under duality [26,28]. The ZXZ Hamiltonian Eq. (1) is dual to two coupled transverse-field Ising chains (on the odd and even sites, respectively): $H'_{\rm SPT} = \lambda_1 \sum_{j=1}^{M-2} \sigma_{2j}^z \sigma_{2j+2}^z + \Gamma \sum_{j=1}^{M} \sigma_{2j}^x + \lambda_2 \sum_{j=1}^{M-3} \sigma_{2j+1}^z \sigma_{2j+3}^z + \Gamma \sum_{j=1}^{M-1} \sigma_{2j+1}^x + \Gamma_2 \sum_{i=1}^{L-1} \sigma_i^x \sigma_{i+1}^x$. When λ_i are the dominant couplings, the SPT phase transforms to the global $\mathbb{Z}_2 \times \mathbb{Z}_2$ *symmetry-broken* phase of the coupled Ising chains. There is an additional $\mathbb{Z}_2$ "swap" symmetry when $\lambda_1 = \lambda_2$, which arises from exchanging the two Ising chains. Excitations of the original SPT correspond to different types of bulk domain walls in the dual symmetry-broken model. When the two chains are decoupled ($\Gamma_2 = 0$), their respective edge spins σ_1^z and σ_2^z are protected from depolarization; however, these operators are not mutually conjugate. But the operator $\sigma_2^z \prod_{i=1}^{M} \sigma_{2i-1}^x$ *is* conjugate to σ_1^z, and it is long-lived, because the product over σ_{2i-1}^x is simply the global spin-flip symmetry G_o on

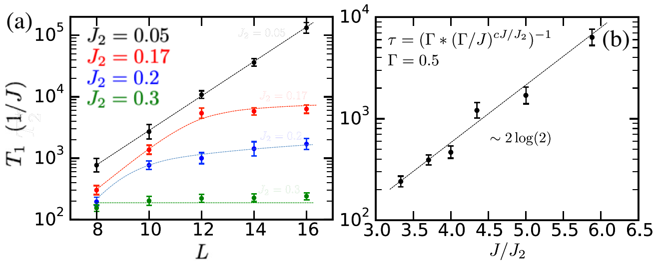


FIG. 2. The depolarization time T_1 of the classical edge bit of the Ising chain Eq. (2). (a) In finite-size chains, T_1 is limited by perturbative processes which flip the spin at opposite ends of the chain. Thus, T_1 increases exponentially with L until it saturates to its infinite-volume limit. (b) The saturated T_1 follows the exponential form e^{cJ/J_2} as predicted by Eq. (3).

odd sites. Under duality, the corresponding long-lived conjugate operators in the SPT are localized on the edge:

$$\begin{array}{ccc} \text{ZXZ} & & \text{Ising} \times \text{Ising} \\ \Sigma^x = \sigma_1^x \sigma_2^z & \longleftrightarrow & \sigma_2^z G_o, \\ \Sigma^y = \sigma_1^y \sigma_2^z & \longleftrightarrow & i\sigma_1^z \sigma_2^z G_o, \\ \Sigma^z = \sigma_1^z & \longleftrightarrow & \sigma_1^z. \end{array} \tag{4}$$

For decoupled transverse-field Ising chains, the depolarization of the edge requires the emission or absorption of a domain wall, which is an off-resonant process. However, if the chains are coupled, the interaction Γ_2 between them can depolarize the edge spins by transforming one type of domain wall into the other. In particular, if $\lambda_1 = \lambda_2$ then there are different types of domain walls with the same energy, and the edge spin can immediately relax via on-shell domain-wall conversion. A similar physical argument explains the lack of a long-lived edge mode in the Potts model [29,30].

As an explicit example of this domain-wall conversion process, consider the following two configurations of spins at the edge of the coupled Ising chains:

$$\begin{array}{ccc} \uparrow\downarrow\downarrow\downarrow\cdots & & \downarrow\downarrow\downarrow\downarrow\cdots \\ & \xRightarrow{\Gamma_2\sigma_1^x\sigma_2^x} & \\ \downarrow\downarrow\downarrow\downarrow\cdots & & \uparrow\downarrow\downarrow\downarrow\cdots. \end{array}$$

Here, the upper (lower) row depicts the odd (even) spins. On the left, there is a single broken Ising bond on the upper chain. The Γ_2 term can hop the broken bond from the upper chain to the lower, flipping both edge spins. When $\lambda_1 = \lambda_2$, there is *no difference* in energy between these two configurations due to the $\mathbb{Z}_2$ swap symmetry. Thus, the protection of the edge spin fails at leading order in perturbation theory, and it quickly depolarizes [Fig. 1(d)].

This suggests that a natural way to restore the edge-spin lifetime is to dimerize the SPT with $\lambda_1 \neq \lambda_2$, which prevents the direct resonant conversion of one domain-wall type to another. Consequently, the autocorrelation times of all the conjugate edge-mode operators Σ^α are exponentially long at infinite temperature, as can be seen in Fig. 1(e), in stark contrast with the transverse-field Ising chain, where only σ_1^z has a long autocorrelation time [Fig. 1(c)]. In the language of quantum information, this means that both depolarization *and* dephasing are strongly suppressed and the edge mode constitutes a coherent qubit.

Interpretation via strong zero modes.—The long-lived edge qubit arises from two *almost* strong edge zero modes (SZMs), which exhibit significant overlap with Σ^α. We briefly summarize the properties of exact and almost SZMs [23,24,31,32]. An exact SZM is an operator which is localized at the edge of the system and commutes with the Hamiltonian up to terms exponentially small in system size. An example is the Majorana zero mode at the edge of the Kitaev chain [33,34].

Such SZMs can be constructed order by order in perturbation theory. For systems with exact SZMs, the perturbative construction converges exponentially and need only be cut off at finite size. In systems with almost SZMs, the same construction produces asymptotic series which must be cut off at some finite order, beyond which the magnitude of the commutator with H increases. This cutoff produces the observed lifetimes in, e.g., Fig. 2(a). These lifetimes are calculated numerically from the time taken for the autocorrelation to reduce by a factor of e.

Returning to the ZXZ model, we attempt to construct two conjugate SZMs by double expansion in Γ and Γ_2, starting from the zeroth-order terms: $\Psi_z^{(0)} = \Sigma^z$ and $\Psi_x^{(0)} = \Sigma^x$. We have explicitly constructed these up to fourth order. We emphasize that the existence of two such *conjugate* SZMs on a single boundary is highly nontrivial and does not occur in either the transverse-field Ising model or the Kitaev chain considered previously [23,24]. Without them, we cannot *locally* encode an edge qubit [35].

The first-order terms are

$$\Psi_z^{(1)} = \frac{\Gamma}{\lambda_1}\sigma_1^x\sigma_2^x\sigma_3^z + \frac{\Gamma_2}{\lambda_1^2 - \lambda_2^2}(\lambda_1\sigma_1^x\sigma_3^z + \lambda_2\sigma_1^y\sigma_2^y\sigma_3^x\sigma_4^z),$$

$$\begin{aligned} \Psi_x^{(1)} = {} & \frac{\Gamma}{\lambda_2}\sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z - \frac{\Gamma_2}{\lambda_1^2 - \lambda_2^2}(\lambda_2\sigma_2^x\sigma_3^x\sigma_4^z + \lambda_1\sigma_1^z\sigma_2^z\sigma_3^z) \\ & + \frac{\Gamma_2\lambda_1}{4\lambda_1^2 - \lambda_2^2}\left(\sigma_1^y\sigma_2^z\sigma_3^y + \left(2\frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1}\right)\sigma_1^x\sigma_2^x\sigma_4^z \right. \\ & \left. - \sigma_1^x\sigma_2^y\sigma_3^y\sigma_4^x\sigma_5^z - 2\frac{\lambda_1}{\lambda_2}\sigma_1^y\sigma_4^y\sigma_5^z\right). \end{aligned}$$

This expression provides insight into the need for dimerization: For $|\lambda_1| = |\lambda_2|$, the expansion breaks down at first order, since $\Psi^{(1)}$ diverges. This leads to a simple prediction: The autocorrelation times of Σ^z and Σ^x should exhibit a dramatic reduction [compared to Fig. 1(e)] when $|\lambda_1|/|\lambda_2| = 1$. This is borne out by our numerics; see Fig. 3.

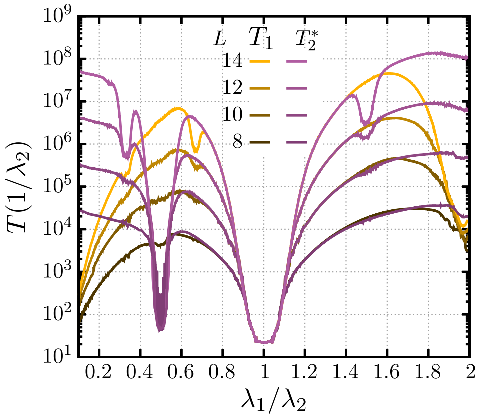


FIG. 3. The decay times T_1 and T_2^* of the autocorrelators of the conjugate edge-operators Σ^z and Σ^x for the ZXZ model, Eq. (1), using exact diagonalization at infinite temperature with $\Gamma = \Gamma_2 = 0.05$. The lifetimes increase exponentially with system size until they saturate due to nearby resonances at rational λ_1/λ_2. The $\lambda_1 = \lambda_2$ resonance is first order, and thus, the edge-operators' lifetime is not enhanced.

More generally, poles in the SZM expansion correspond to physical resonances where the lifetimes becomes short. These can correspond to complicated physical processes. For example, there is an additional pole in $\Psi_x^{(1)}$ at $2|\lambda_1| = |\lambda_2|$ but not in $\Psi_z^{(1)}$. This corresponds to the large dip in the coherence time of Σ^x around $\lambda_1 = \lambda_2/2$ in Fig. 3. Physically, a broken bond on the edge of the lower chain can hop into the bulk of the upper chain, creating *two* broken bonds. This process does not change the energy if $\lambda_1 = \lambda_2/2$. However, if a broken bond is on the edge of the upper chain, the Γ_2 coupling can only move it to the edge of the lower chain, so there is no corresponding dip in the coherence time of Σ^z.

At second order in the SZM expansion, further poles appear at $|\lambda_1|/|\lambda_2| = ⅓, 2$ for Σ^z, and at ½, ³⁄₂ for Σ^x, the effects of which are visible in Fig. 3. In general, suppose $|\lambda_1|/|\lambda_2| = p/q$ for integers p and q without common prime factors. There is a resonance if the change in energy from flipping an edge spin under the diagonal part of the Hamiltonian ($\Gamma = \Gamma_2 = 0$) can be matched by flipping bulk spins. In the case of coupled Ising chains, the dangerous processes involve flipping either edge spin, individually or together. Let us factor the change in energy due to flipping a spin at site j as $2\lambda_1\Delta_j/p$. At the edge, $\Delta_1 = \pm p$ and $\Delta_2 = \pm q$. In the bulk, the sum of Δ_j for $j > 2$ is an arbitrary linear combination of $2p$ and $2q$, which has even parity. Thus, when p is even, the edge-spin flip cost Δ_1 can be canceled by the sum of bulk flips Δ_j, and Σ^z has a resonance. On the other hand, p even forces q odd, so Σ^x does not have a resonance. If q is even instead, the reverse is true. When *p and q* are odd, the cost of flipping both edge spins $\Delta_1 + \Delta_2$ is even, so both edge operators suffer a resonance at the same order.

This analysis shows that there are no divergence-free, rational $|\lambda_1|/|\lambda_2|$ for both conjugate edge operators. Nevertheless, for large p and q, the resonances only occur at high orders in perturbation theory, and the coherence time is significantly enhanced, as emphasized in Fig. 3. One might be tempted to bypass resonances by choosing incommensurate λ_1 and λ_2, where there are formal results on bulk prethermal behavior [36]. However, the poles due to nearby resonances always produce large coefficients in the SZM expansion at sufficiently high order. Physically, the linewidth of the high-order resonances, clear in Fig. 3, may be interpreted in terms of the energy uncertainty of the domain walls involved in the processes that flip the edge spin [23].

Experimental realization.—A direct experimental realization of our proposal can be implemented in a 1D optical-tweezer array (Fig. 1) of single alkali or alkaline-earth atoms [37–41]. Such systems have emerged as powerful platforms for building up many-body quantum systems atom by atom. Here, we envision the effective spin degree of freedom in Eq. (1) to be formed by two hyperfine atomic ground states. The most natural Hamiltonian in such systems is a long-range transverse-field Ising model, with power-law interactions decaying as $1/r^6$. The physics underlying the SZM is robust to the presence of long-range interactions, a feature we explore in detail in the Supplemental Material [26]; we note that this contrasts with the Floquet SPT MBL-based strategy for realizing an infinite-temperature edge qubit previously proposed [42]. Since the power-law tail does not qualitatively change the system's behavior, here, we neglect it for simplicity and consider the Hamiltonian $H = \sum_{i=1}^{N} h_i\sigma_i^x + \sum_{i=1}^{N-1}\lambda_i\sigma_i^z\sigma_{i+1}^z$. The Ising interaction is generated by dressing the ground hyperfine state with an excited Rydberg state using a far-detuned laser [43–47]; the resulting Rydberg blockade induces strong effective spin-spin interactions with a range on the order of a few microns [48]. The transverse field is implemented by resonant Raman coupling. Finally, by using techniques from Floquet engineering, it is possible to realize the dimerized ZXZ Hamiltonian stroboscopically [42,49]. By periodically modulating the Ising coupling as $\omega\cos(\omega t)\lambda_i\sigma_i^z\sigma_{i+1}^z$ [50], one generates (at leading order in a Floquet-Magnus expansion) dynamics that are governed by an effective Floquet Hamiltonian [42]:

$$H_F = \sum_{i=1}^{N} h_i a(\lambda_1,\lambda_2)\sigma_i^x - \sum_{i=2}^{N-1} h_i b(\lambda_1,\lambda_2)\sigma_{i-1}^z\sigma_i^x\sigma_{i+1}^z, \quad (5)$$

where $a(\lambda_1,\lambda_2) = \frac{1}{2}[J_0(2(\lambda_1-\lambda_2)) + J_0(2(\lambda_1+\lambda_2))]$, $b(\lambda_1,\lambda_2) = J_0(2(\lambda_1-\lambda_2)) - a(\lambda_1,\lambda_2)$, and $J_0(x)$ is a Bessel function of the first kind [42,51]. Dimerization of the transverse-field h_i is inherited by the ZXZ coupling in the Floquet Hamiltonian.

The relatively large interatom spacing of a typical Rydberg tweezer array enables experiments to probe the long lifetime associated with our proposed edge qubit. As a concrete protocol, (1) initialize the Rydberg spin chain in a randomly oriented product state (i.e., effectively at infinite temperature), (2) projectively initialize the edge qubit by measuring Σ^z, (3) evolve the system for time t, and (4) measure Σ^z again to obtain the correlation function $\langle\Sigma^z(t)\Sigma^z(0)\rangle$ after averaging over multiple runs. The decay of the $\langle\Sigma^z(t)\Sigma^z(0)\rangle$ correlator measures the lifetime of z polarization. To demonstrate a coherent quantum bit, one must perform the same procedure for Σ^x. In the language of atomic spectroscopy, this is analogous to performing a Ramsey sequence to probe the dephasing time T_2^* of the qubit, while the Σ^z autocorrelator probes the depolarization time T_1.

To ensure that there are parameters for the Floquet Hamiltonian Eq. (5) that realize a dimerized ZXZ model with long edge coherence, we have numerically simulated the Floquet Hamiltonian for an $L = 14$ spin chain [26]. In addition to demonstrating that our proposal works for a broad range of parameters, our numerics indicate that the difference between edge- and bulk-spin autocorrelators can be distinguished on timescales much shorter than the typical lifetime of the Rydberg-dressed state [52]. Moreover, previous work on the transverse-field Ising model has also found that the enhancement of the edge depolarization time (compared to the bulk) can survive external noise, a feature we expect to extend to the boundary qubit in the cluster model [32].

Our work opens the door to a number of intriguing future directions. First, by exploring other symmetry regimes, higher spatial dimensions, and models with unbounded local Hilbert spaces, it may be possible to extend our mechanism for edge-mode stability to more generic settings [53,54]. Second, while we have focused on an experimental proposal based on Floquet engineering, it would be interesting to investigate the prethermal dynamics of a coupled 1D Rydberg ladder; this geometry exhibits the same symmetries as the ZXZ model and thus might provide a simpler route to realizing symmetry-enhanced edge modes. Finally, building on techniques developed in the context of many-body localized SPT phases [14,55], it would be interesting to explore hybrid quantum-information protocols where symmetry-enhanced edge qubits play the role of robust quantum memories.

The authors would like to thank Soonwon Choi, Paul Fendley, Francisco Machado, Johannes Zeiher, and Immanuel Bloch for stimulating discussions. This work was supported by the NSF through Grant No. PHY-1752727 and through the QII-TAQS program (Grant No. 1936100), the ARO through the MURI program (Grant No. W911NF-17-1-0323), the David and Lucile Packard foundation, and the W. M. Keck foundation. This work was performed in part at the Aspen Center for Physics, which is supported by National Science Foundation Grant No. PHY-1607611, and at the KITP supported by Grant No. NSF PHY-1748958. Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

Note added.—A similar model with $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT order was considered by Parker *et al.* [56]; however, they focus on the effects of the SPT phase on a single SZM in a nearby proximate symmetry-broken phase, rather than considering the conjugate edge modes in the SPT itself.

[1] F. Pollmann, A. M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B **81**, 064439 (2010).
[2] L. Fidkowski and A. Kitaev, Phys. Rev. B **83**, 075103 (2011).
[3] X. Chen, Z.-C. Gu, Z.-X. Liu, and X.-G. Wen, Science **338**, 1604 (2012).
[4] M. Levin and Z.-C. Gu, Phys. Rev. B **86**, 115109 (2012).
[5] X. Chen, Z.-C. Gu, Z.-X. Liu, and X.-G. Wen, Phys. Rev. B **87**, 155114 (2013).
[6] O. Viyuela, A. Rivas, and M. A. Martin-Delgado, Phys. Rev. Lett. **112**, 130401 (2014).
[7] Y.-Z. You and C. Xu, Phys. Rev. B **90**, 245120 (2014).
[8] T. Senthil, Annu. Rev. Condens. Matter Phys. **6**, 299 (2015).
[9] R. Nandkishore and D. A. Huse, Annu. Rev. Condens. Matter Phys. **6**, 15 (2015).
[10] D. A. Abanin and Z. Papić, Ann. Phys. (Berlin) **529**, 1700169 (2017).
[11] A. Chandran, V. Khemani, C. R. Laumann, and S. L. Sondhi, Phys. Rev. B **89**, 144201 (2014).
[12] A. C. Potter and A. Vishwanath, arXiv:1506.00592.
[13] Y. Bahri, R. Vosk, E. Altman, and A. Vishwanath, Nat. Commun. **6**, 7341 (2015).
[14] N. Y. Yao, C. R. Laumann, and A. Vishwanath, arXiv:1508.06995.
[15] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Science **349**, 842 (2015).
[16] J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Science **352**, 1547 (2016).
[17] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, Nat. Phys. **12**, 907 (2016).
[18] H. P. Lüschen, P. Bordia, S. S. Hodgman, M. Schreiber, S. Sarkar, A. J. Daley, M. H. Fischer, E. Altman, I. Bloch, and U. Schneider, Phys. Rev. X **7**, 011034 (2017).
[19] P. Bordia, H. Lüschen, U. Schneider, M. Knap, and I. Bloch, Nat. Phys. **13**, 460 (2017).
[20] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[21] W. Son, L. Amico, R. Fazio, A. Hamma, S. Pascazio, and V. Vedral, Europhys. Lett. **95**, 50001 (2011).
[22] X. Chen, Y.-M. Lu, and A. Vishwanath, Nat. Commun. **5**, 3507 (2014).

[23] J. Kemp, N. Y. Yao, C. R. Laumann, and P. Fendley, J. Stat. Mech. (2017) 063105.
[24] D. V. Else, P. Fendley, J. Kemp, and C. Nayak, Phys. Rev. X **7**, 041062 (2017).
[25] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Phys. Rev. Lett. **59**, 799 (1987).
[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.200506 for technical details on the cluster model [27] and further numerical simulations of the experimental proposal.
[27] D. Abanin, W. De Roeck, W. W. Ho, and F. Huveneers, Commun. Math. Phys. **354**, 809 (2017).
[28] T. Kennedy and H. Tasaki, Phys. Rev. B **45**, 304 (1992).
[29] A. S. Jermyn, R. S. K. Mong, J. Alicea, and P. Fendley, Phys. Rev. B **90**, 165106 (2014).
[30] N. Moran, D. Pellegrino, J. K. Slingerland, and G. Kells, Phys. Rev. B **95**, 235127 (2017).
[31] P. Fendley, J. Phys. A **49**, 30LT01 (2016).
[32] L. M. Vasiloiu, F. Carollo, and J. P. Garrahan, Phys. Rev. B **98**, 094308 (2018).
[33] A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).
[34] P. Fendley, J. Stat. Mech. (2012) P11020.
[35] In the Kitaev chain, a qubit can be constructed from SZMs, but it must be encoded nonlocally using the Majorana zero modes on both ends of the chain.
[36] D. V. Else, W. W. Ho, and P. T. Dumitrescu, Phys. Rev. X **10**, 021032 (2020).
[37] D. Barredo, S. De Léséleuc, V. Lienhard, T. Lahaye, and A. Browaeys, Science **354**, 1021 (2016).
[38] M. Endres, H. Bernien, A. Keesling, H. Levine, E. R. Anschuetz, A. Krajenbrink, C. Senko, V. Vuletic, M. Greiner, and M. D. Lukin, Science **354**, 1024 (2016).
[39] M. A. Norcia, A. W. Young, and A. M. Kaufman, Phys. Rev. X **8**, 041054 (2018).
[40] A. Cooper, J. P. Covey, I. S. Madjarov, S. G. Porsev, M. S. Safronova, and M. Endres, Phys. Rev. X **8**, 041055 (2018).
[41] S. Saskin, J. T. Wilson, B. Grinkemeyer, and J. D. Thompson, Phys. Rev. Lett. **122**, 143002 (2019).
[42] I.-D. Potirniche, A. C. Potter, M. Schleier-Smith, A. Vishwanath, and N. Y. Yao, Phys. Rev. Lett. **119**, 123601 (2017).
[43] R. M. W. van Bijnen and T. Pohl, Phys. Rev. Lett. **114**, 243002 (2015).
[44] A. W. Glaetzle, M. Dalmonte, R. Nath, C. Gross, I. Bloch, and P. Zoller, Phys. Rev. Lett. **114**, 173002 (2015).
[45] E. A. Goldschmidt, T. Boulier, R. C. Brown, S. B. Koller, J. T. Young, A. V. Gorshkov, S. L. Rolston, and J. V. Porto, Phys. Rev. Lett. **116**, 113001 (2016).
[46] J. Zeiher, P. Schauß, S. Hild, T. Macri, I. Bloch, and C. Gross, Phys. Rev. X **5**, 031015 (2015).
[47] J. Zeiher, J.-y. Choi, A. Rubio-Abadal, T. Pohl, R. van Bijnen, I. Bloch, and C. Gross, Phys. Rev. X **7**, 041063 (2017).
[48] In the blockage regime, the effective interaction strength scales as $\sim[(\Omega^4)/(8\Delta^3)][(1)/(1+|r_i-r_j|^6/R_c^6)]$, where the interaction range $R_c=(-C_6/\Delta)^{1/6}$ depends on the van der Waals coefficient C_6 of the Rydberg-Rydberg interaction, Ω is the driving Rabi frequency, and Δ the associated detuning.
[49] T. Iadecola, L. H. Santos, and C. Chamon, Phys. Rev. B **92**, 125107 (2015).
[50] Note that any driving function which changes sign every half-period will suffice. In an experiment, perhaps the simplest approach to realizing such a modulated Ising coupling is to alternate the sign of the detuning between the two halves of the drive period.
[51] We note that as written, Eq. (5) is dual to free fermions and would thus trivially exhibit exact SZMs, even undimerized. In order to avoid this, we add interaction terms of the form $V\sum\sigma_i^x\sigma_{i+1}^x$ to the time-dependent transverse-field Ising Hamiltonian and propagate this through to the effective Hamiltonian before performing our numerical simulations.
[52] J. Lee, M. J. Martin, Y.-Y. Jau, T. Keating, I. H. Deutsch, and G. W. Biedermann, Phys. Rev. A **95**, 041801(R) (2017).
[53] D. V. Else, P. Fendley, J. Kemp, and C. Nayak, Phys. Rev. X **7**, 041062 (2017).
[54] F. Machado, D. V. Else, G. D. Kahanamoku-Meyer, C. Nayak, and N. Y. Yao, Phys. Rev. X **10**, 011043 (2020).
[55] S. Choi, N. Y. Yao, S. Gopalakrishnan, and M. D. Lukin, arXiv:1508.06992.
[56] D. E. Parker, R. Vasseur, and T. Scaffidi, Phys. Rev. Lett. **122**, 240605 (2019).